# Fluctuations and Correlations: Lattice QCD vs. HRG

Prasad Hegde (BNL)

[for the HotQCD Collaboration]

4th October 2011.

Fluctuations, Correlations and the
RHIC Low-Energy Runs, Oct 3-5
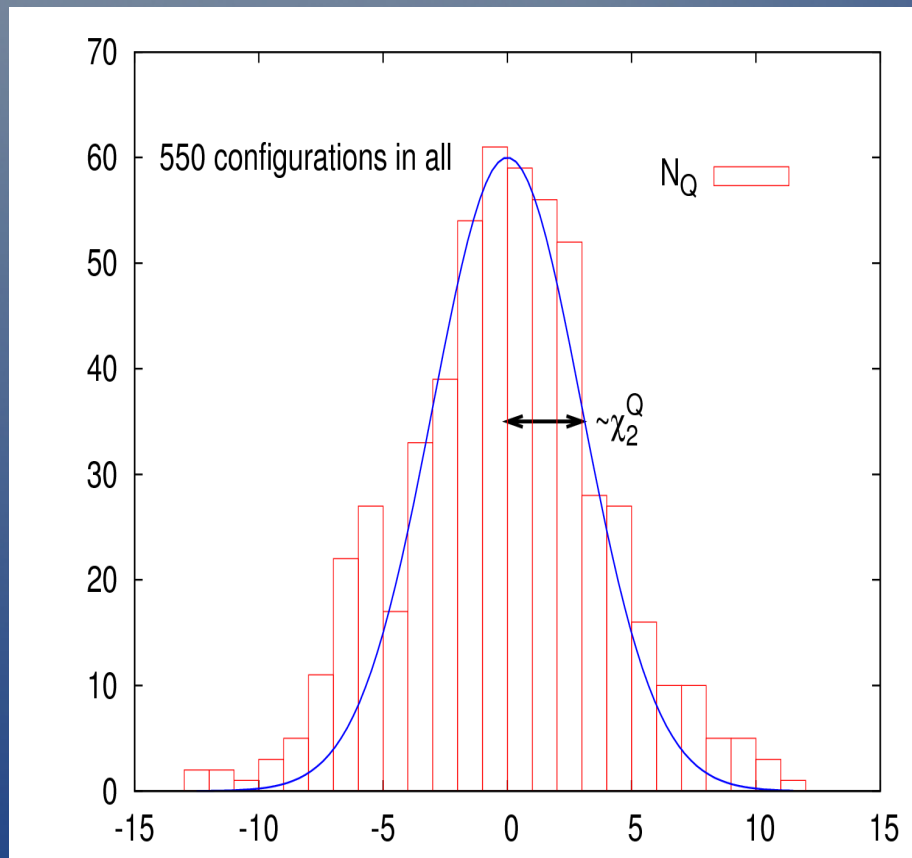
# The HOTQCD Collaboration

- A. Bazavov (BNL)
- T. Bhattacharya (LANL)
- M. Cheng (LLNL)
- N. Christ (Columbia)
- C. deTar (Utah)
- H.-T. Ding (BNL)
- S. Gottlieb (Indiana)
- R. Gupta (Los Alamos)
- P. Hegde (BNL)
- U. Heller (APS)
- C. Jung (BNL)
- F. Karsch (BNL-Bielefeld)
- E. Laermann (Bielefeld)

- L. Levkova (Utah)
- R. Mawhinney (Columbia)
- S. Mukherjee (BNL)
- P. Petreczky (BNL)
- D. Renfrew (BNL)
- C. Schmidt (Bielefeld)
- R. Soltz (LLNL)
- W. Soeldner (Regensburg)
- R. Sugar (UCSB)
- D. Toussaint (Arizona)
- W. Unger (ETH)
- P. Vranas (LLNL)

# Outlook

- Present results for the lowest-order susceptibilities.
  - (For results on higher orders, see talk by C. Schmidt this afternoon.)

- Discuss the systematics involved in measurement and extrapolating to the continuum.

- Compare our results with the Hadron Resonance Gas model (HRG).

# Quark Number Susceptibilities

$$\frac{P(\mu_B, \mu_Q, \mu_S)}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \frac{\chi_{ijk}^{BQS}}{T^{i+j+k}} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$



550 configurations in all

$N_Q$

$\sim \chi_2^Q$

- Measure moments of the corresponding charge distributions.

- Off-diagonals measure correlations between conserved charges.

# Susceptibilities on the Lattice

- HTL expressions for the lowest susceptibilities known. These hold for temperatures above ~ 1.2 $T_c$.

- For smaller T and higher orders, the lattice is the only way forward.

- Chemical potential on the lattice:

$$\sum_x \bar{\psi}_x \gamma_4 \left[ \mathrm{e}^{\mu} U_4(x) \psi_{x+\hat{4}} - \mathrm{e}^{-\mu} U_4^{\dagger}(x) \psi_{x-\hat{4}} \right]$$

- Non-hermitian for $\mu \neq 0$, but derivatives at $\mu=0$ measurable.

# Staggered Fermions

- We computed $\chi_2$, $\chi_4$ and $\chi_6$ as well as off-diagonals using a version of the staggered action i.e. the HISQ action.

- Staggered fermions are:

    - Inexpensive to simulate.

    - Unfortunately, not easy to take the continuum limit.

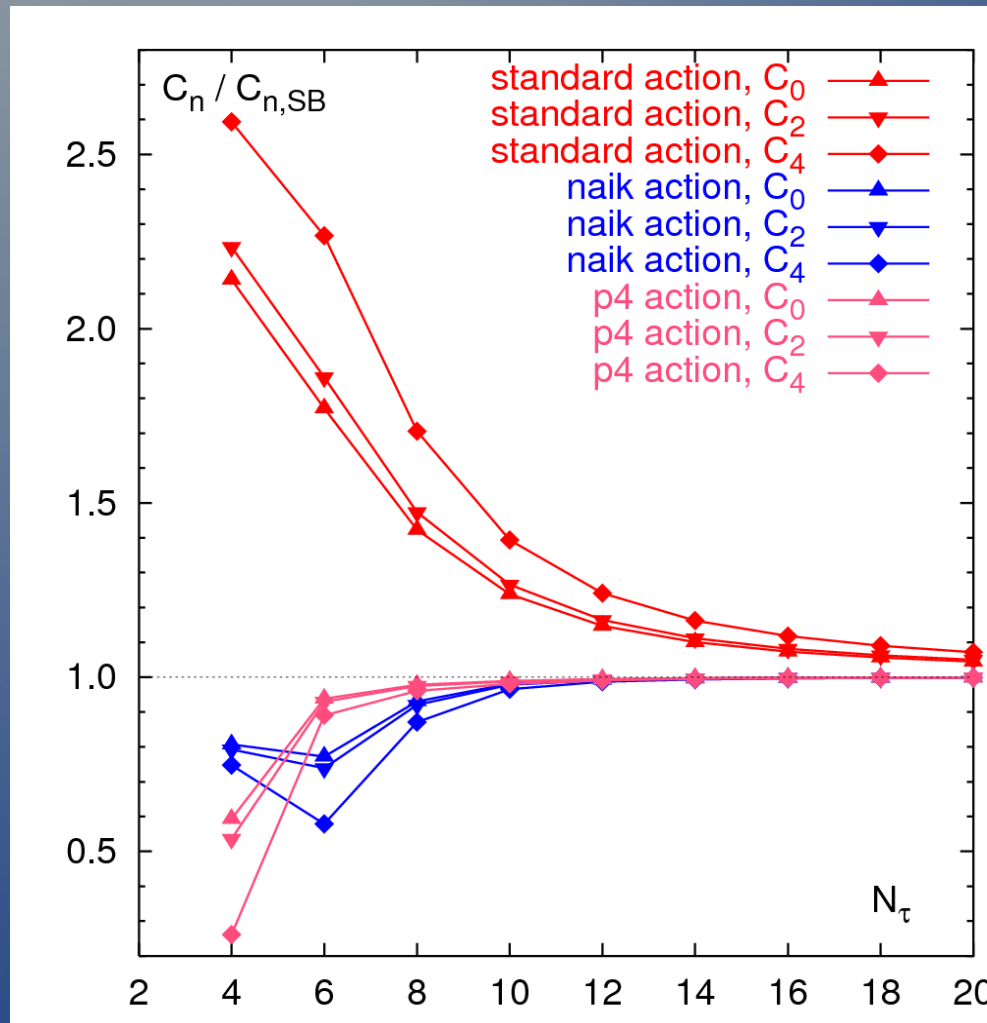    - Suffer from the doubling problem: Sixteen pions instead of one, with exact degeneracy only in the continuum.

# Taste Breaking
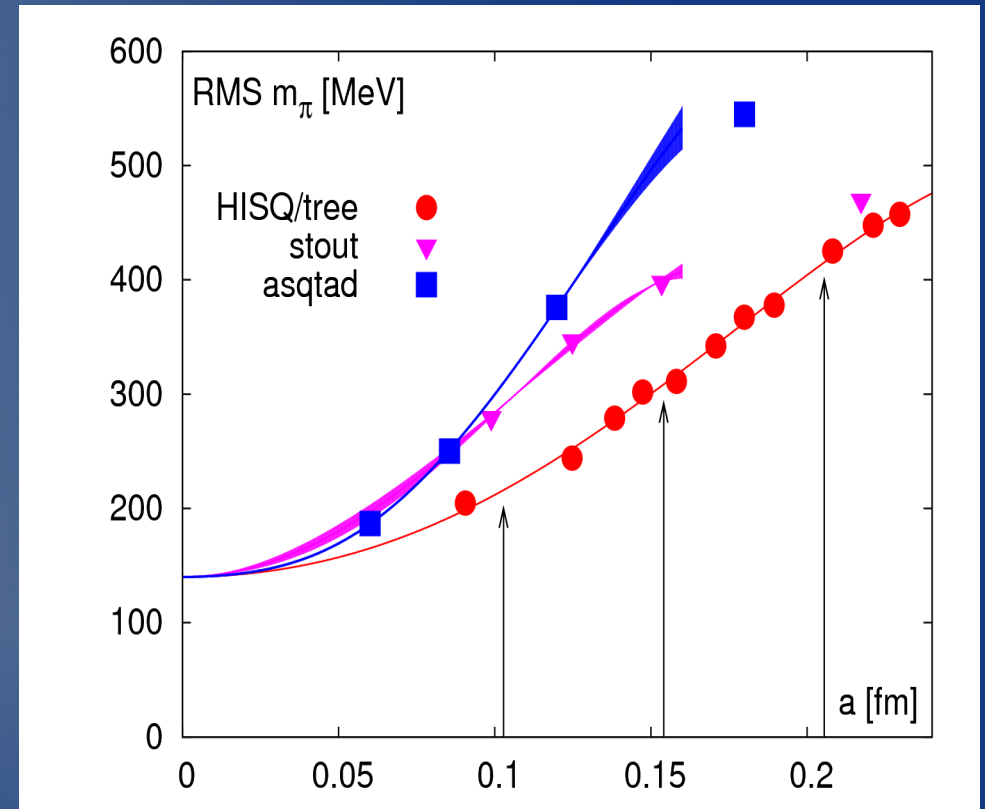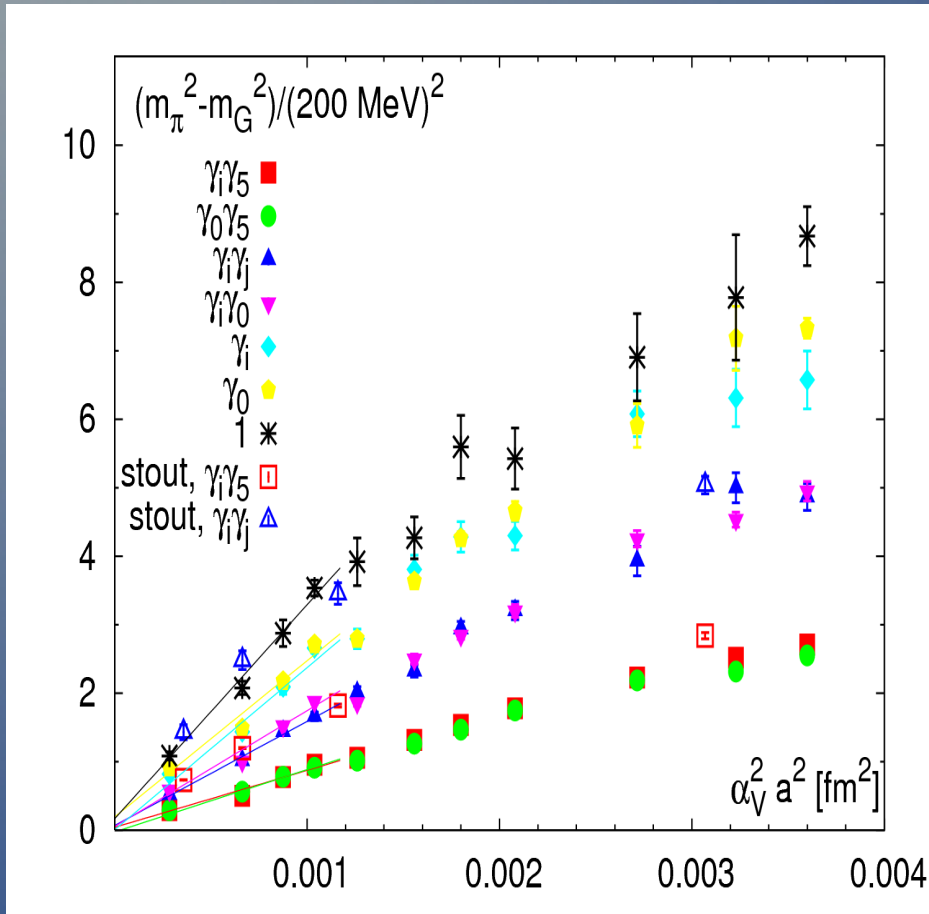


A. Bazavov et al. [HotQCD], PoS LATTICE 2010 (2010), 169.

- Taste-breaking ~ $(\alpha_V a)^2$.

- Speak of RMS pion mass: Approx. 220 MeV at $N_t$=12.
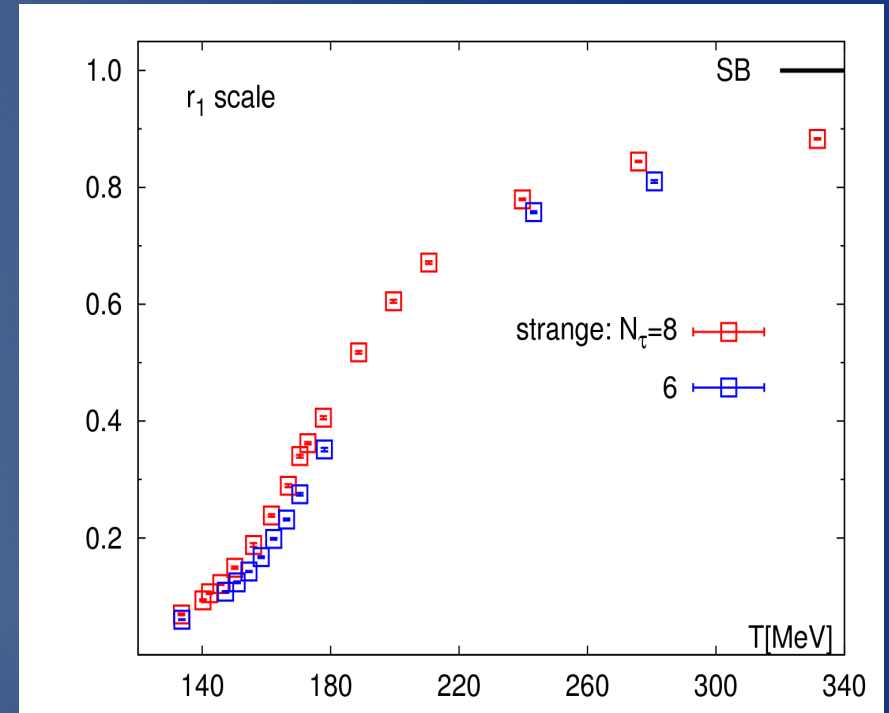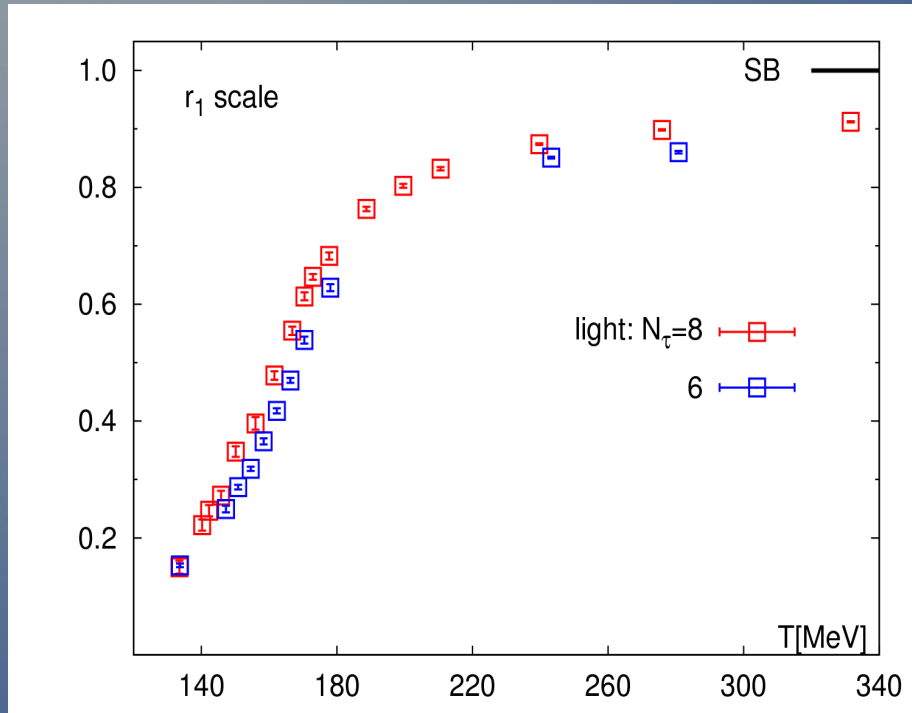
# Modifications to the Dispersion Relation



- Cutoff errors present even in the free gas limit.

- $O(a^2)$ errors with the standard staggered action; deviation by ~50% at $N_t=8$.

- $O(a^4)$ (<10%) for improved actions (HISQ, p4).

C. Allton et al. Phys.Rev. D66 (2002) 074507

# Susceptibilities: A First Look

Caveat: All figures presented here are HOTQCD preliminary!!

# Susceptibilities: A First Look



- Deviation from ideal gas by ~10% at highest T studied.
- Cutoff dependence seen in going from $N_t$ = 6 to 8.

# Setting the Scale

- Scale varies by a few MeV depending upon the observable chosen.

- One generally uses the static quark potential ($r_0$ or $r_1$).

- All choices lead to the same value in the continuum, choose observable with the least cutoff dependence.

- We found a smaller cutoff dependence with the kaon decay constant ($f_K$).

# Choice of Observable: $r_1$ vs. $f_K$
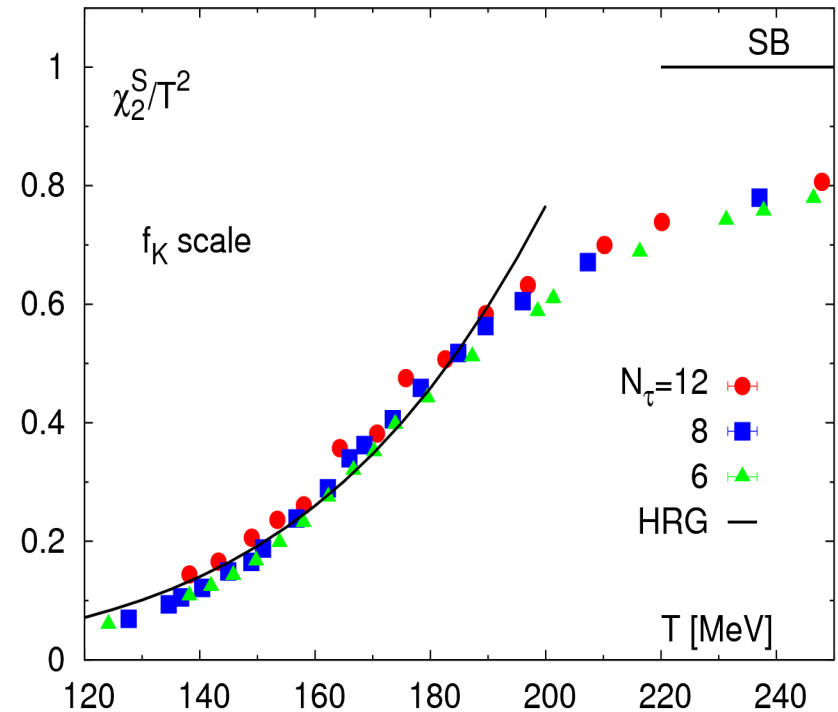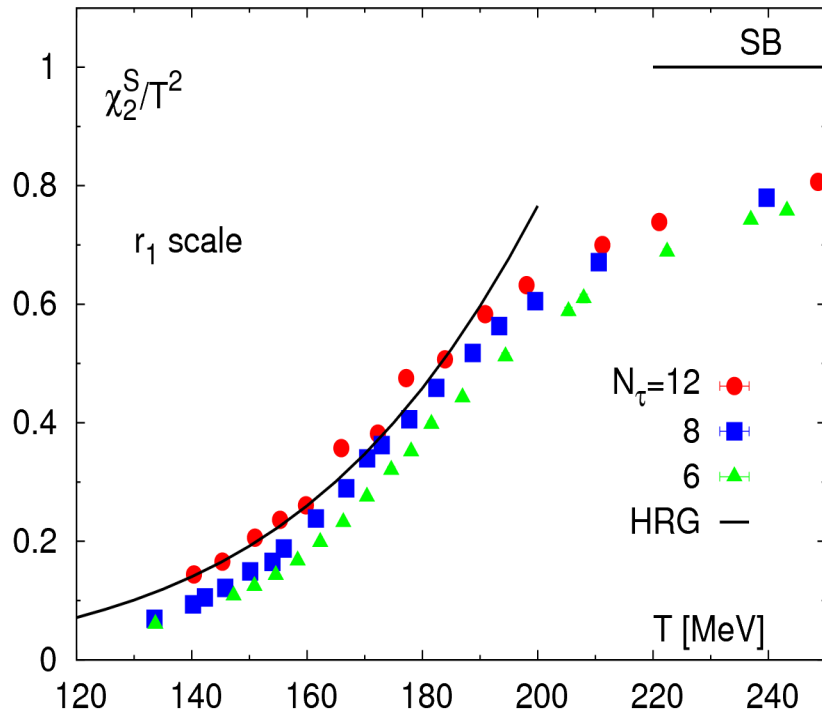


- Smaller cutoff dependence when $f_K$ is used to set the scale.

# Consistency between $r_1$ and $f_K$
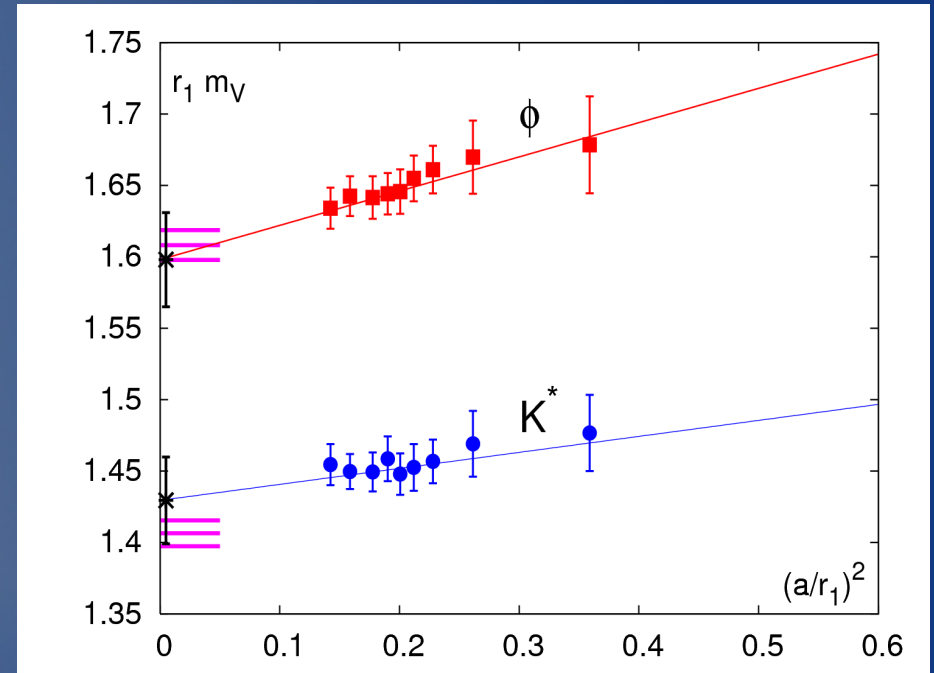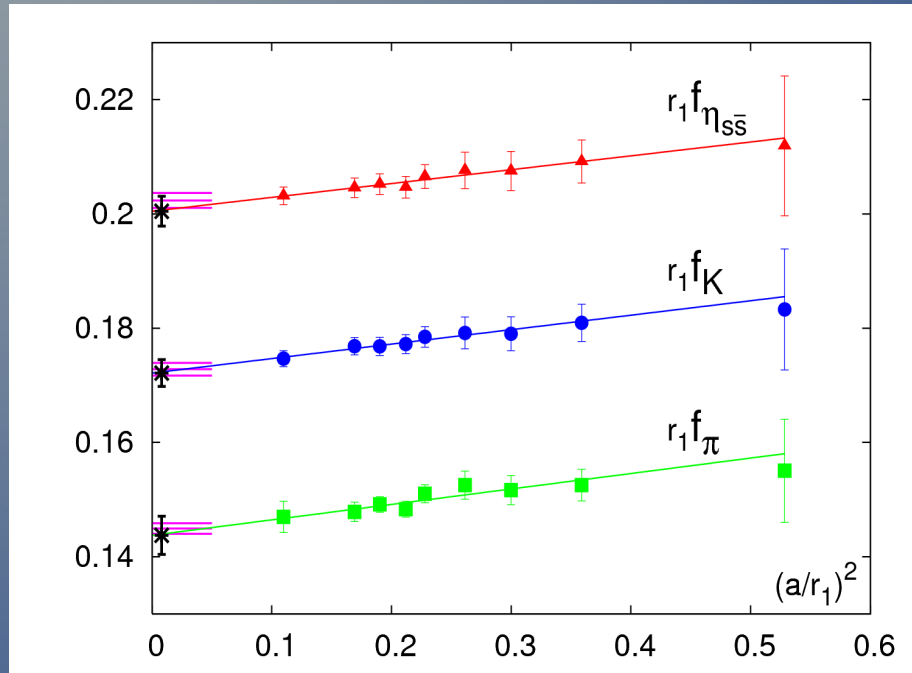


- The difference in T between the two scales is about 2 MeV at $N_t=12$.

- The continuum extrapolations done with either observable are consistent.

# Hadron Resonance Gas

$$\frac{P}{T^4} = \frac{1}{\pi^2} \sum_i d_i K_2\left(\frac{m_i}{T}\right)\left(\frac{m_i}{T}\right)^2 \cosh\left[B_i\mu_B + Q_i\mu_Q + S_i\mu_S\right]$$

- We have included all resonances upto ~ 2.5 GeV.

- Satisfactory description for temperatures below the transition.

- P,$\chi_n$ etc. increase monotonically with T.

- Deviations from HRG would constitute a signal for non-trivial dynamics.

# Baryon Number Susceptibility
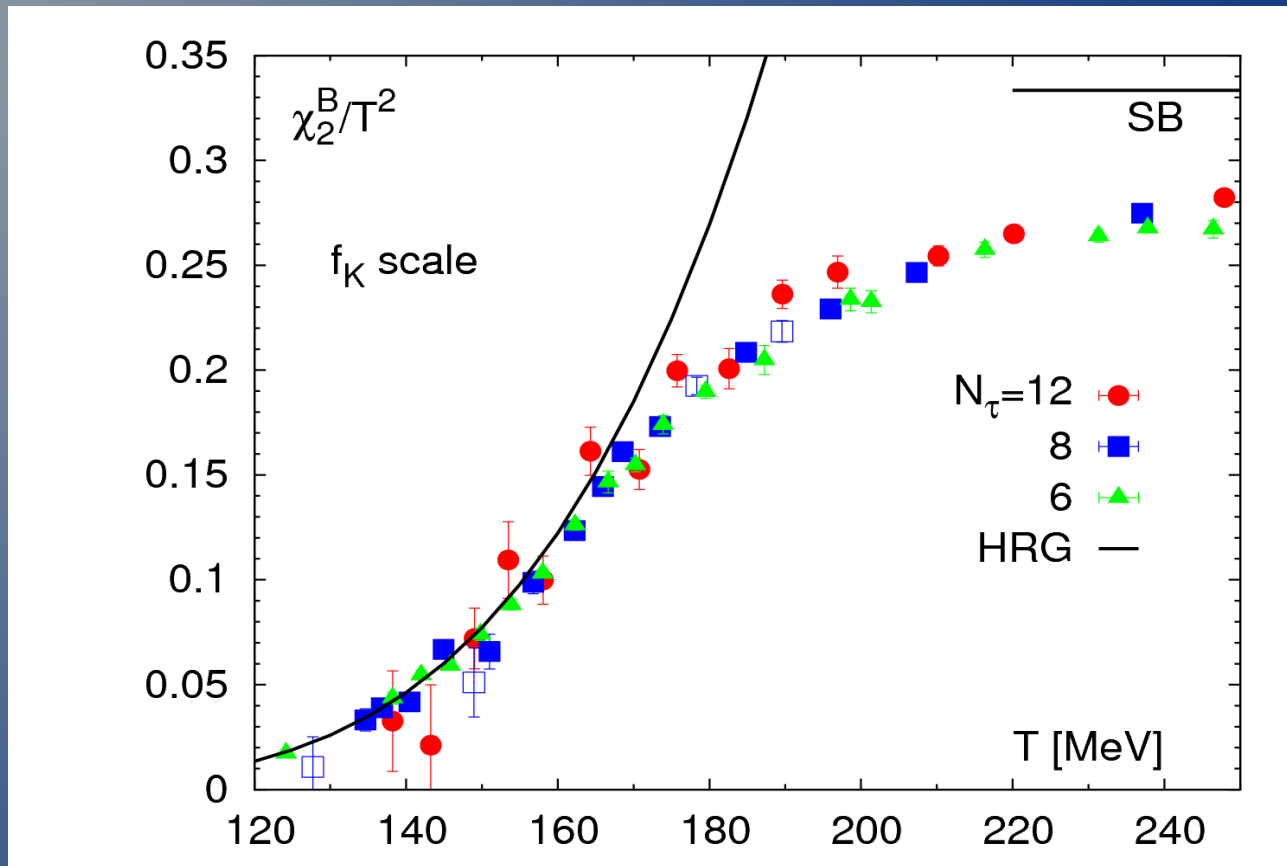


- Signal dominated by protons in the low-T phase.
- SB limit equal to 1/3: Ideal gas of u,d and s quarks.

# Electric Charge Susceptibility
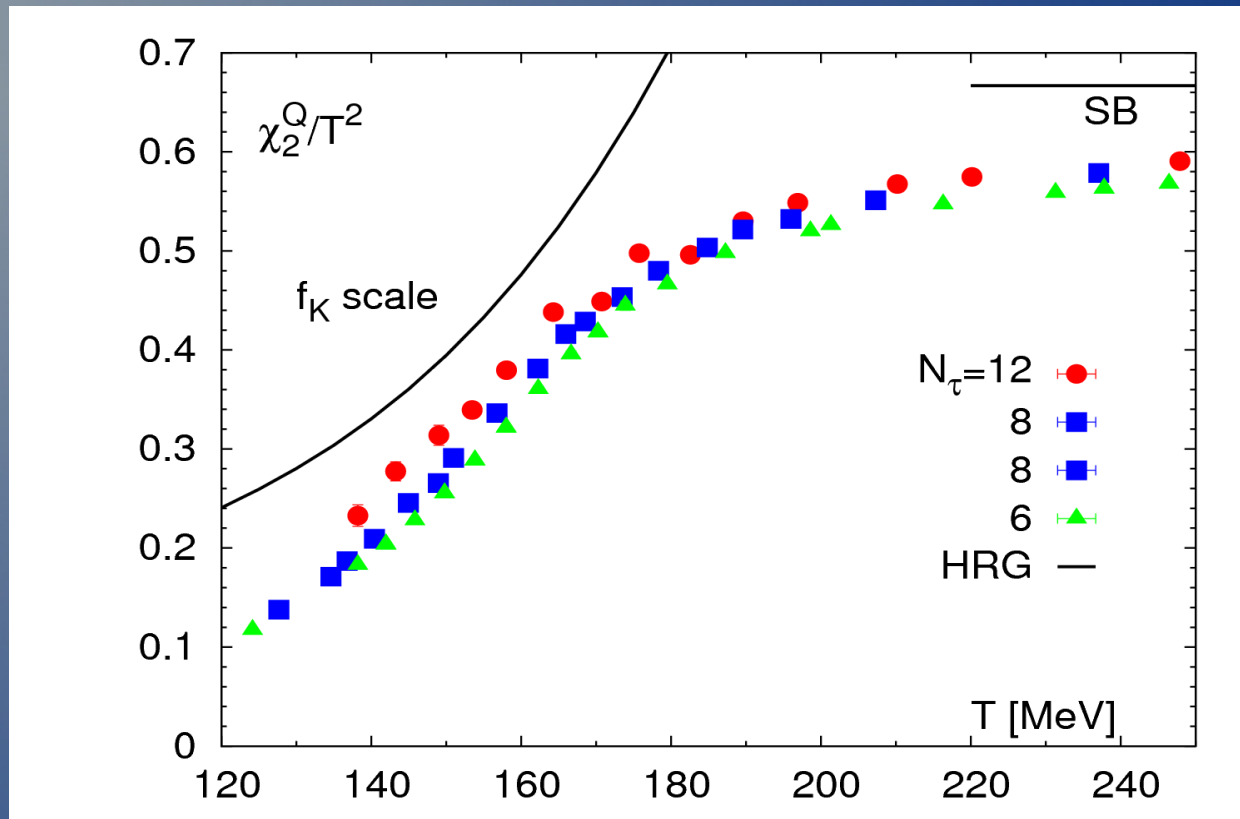


- Signal dominated by pions at low temperatures.

- Poor agreement with Hadron Resonance Gas, but the disagreement is known to arise due to taste violations.
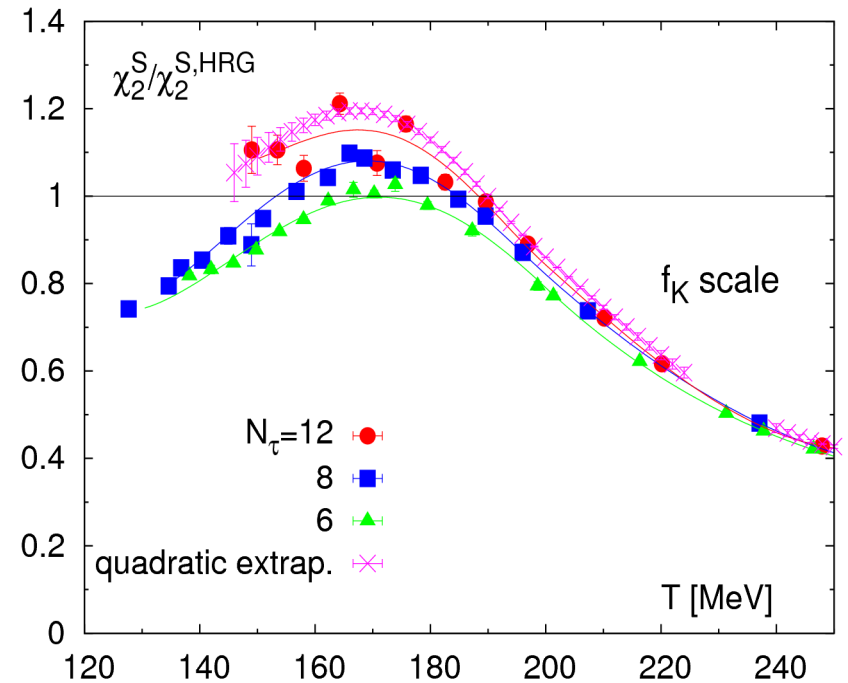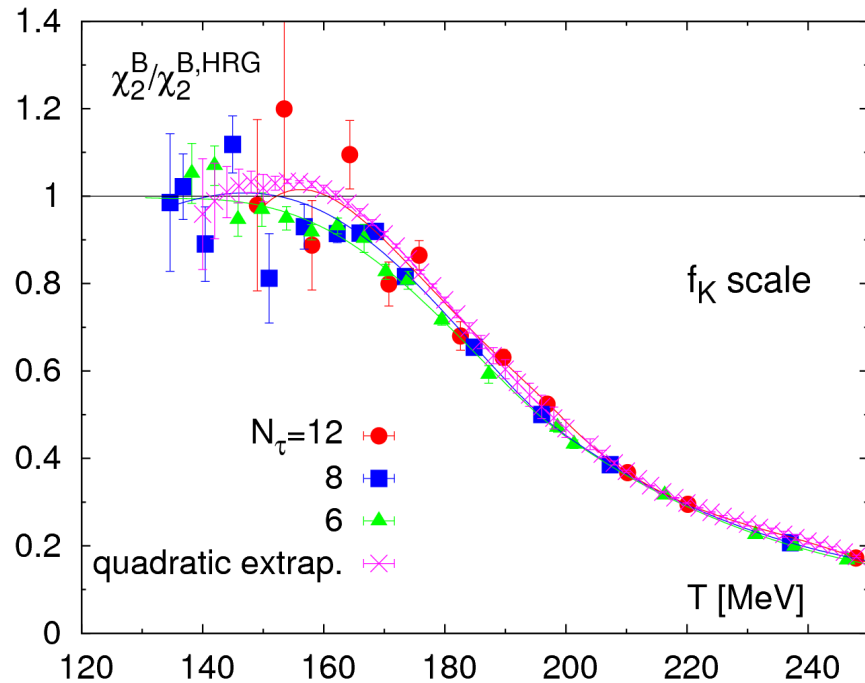
# Continuum Extrapolations

- We also attempted a continuum extrapolation.

- We assumed a polynomial scaling ansatz viz.

$$\mathcal{O}(N_\tau) = \mathcal{O}_{\text{cont.}} + \frac{A}{N_\tau^2}$$

- The extrapolated results were compared to the HRG predictions for T below ~160 MeV.

- Good agreement for baryon number and strangeness.

- Electric charge sensitive to the value of $m_\pi$.

# Baryon Number and Strangeness



- Strangeness exceeds HRG predictions. For baryon number, HRG predicts that ratio of cumulants should be unity. This holds from T < ~170 MeV.

# Charge Susceptibility vs. HRG
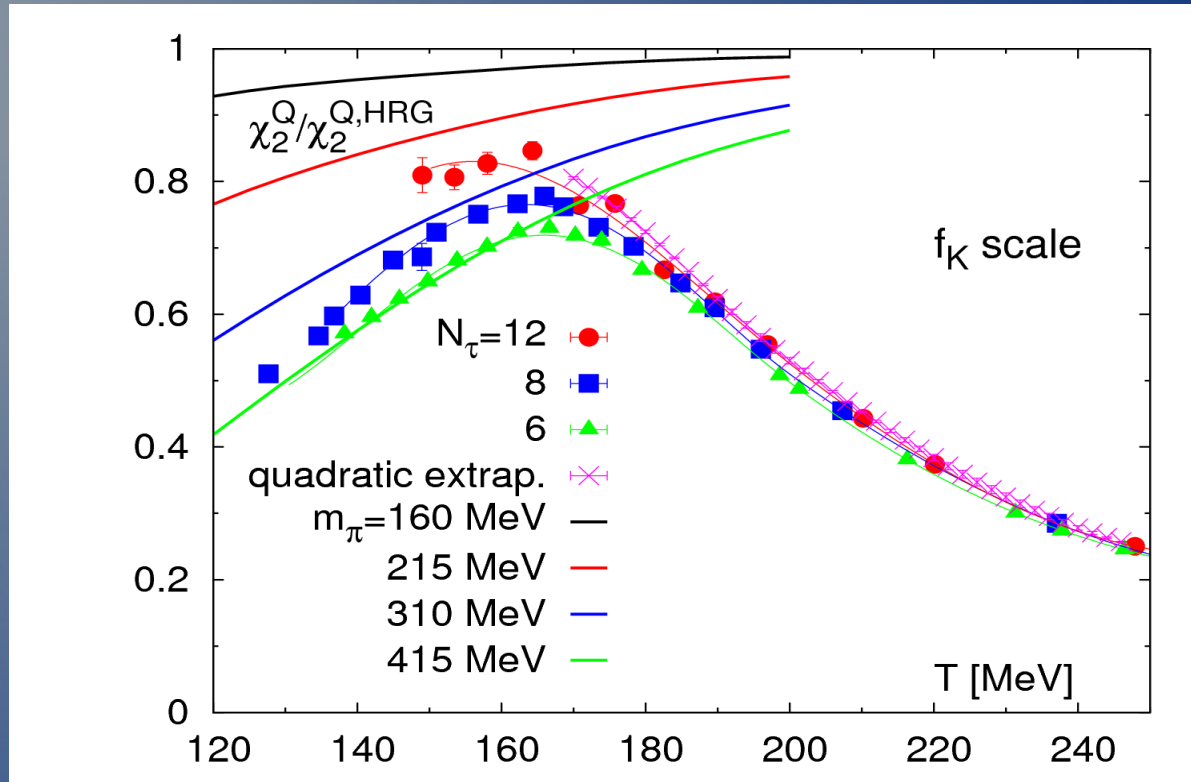


- The HRG curves for Q are very sensitive to the pion mass used.

- Including complete taste spectrum in HRG improves the agreement (P. Petreczky and P. Huovinen, Nucl. Phys. A837 (2010), 26.)

# Off-Diagonals and Ratios
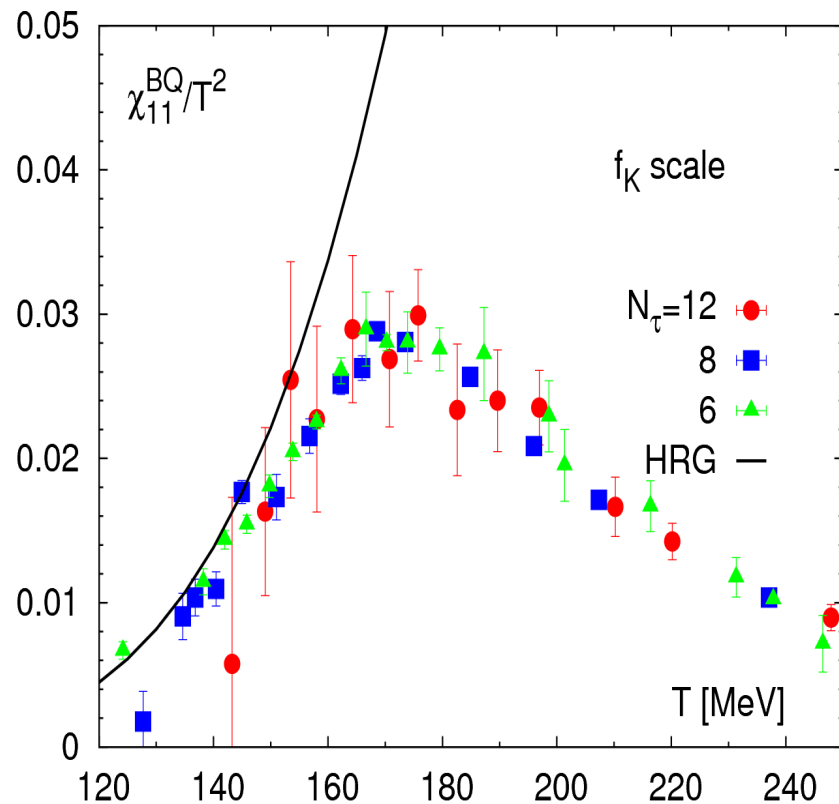


- **Off-diagonals measure correlations between different flavors/charges.**

- **$\chi^{BQ}$ dominated by protons at low T.**

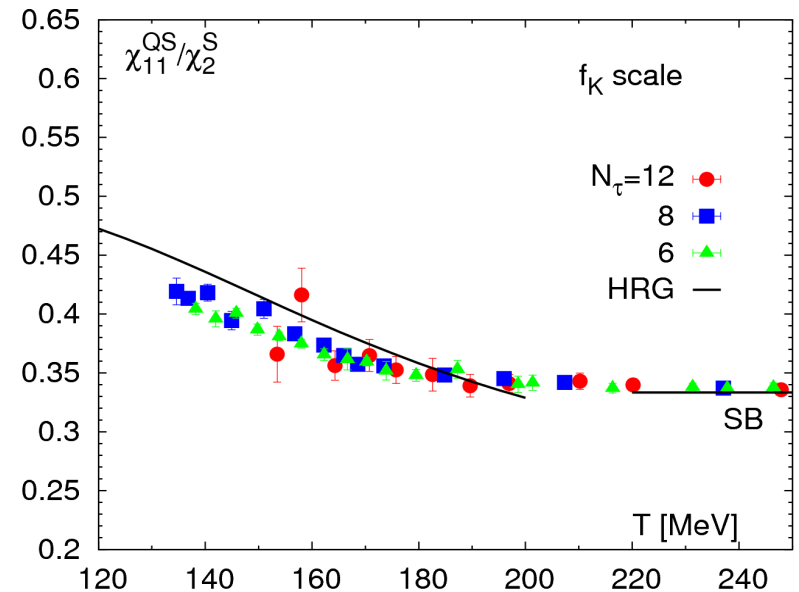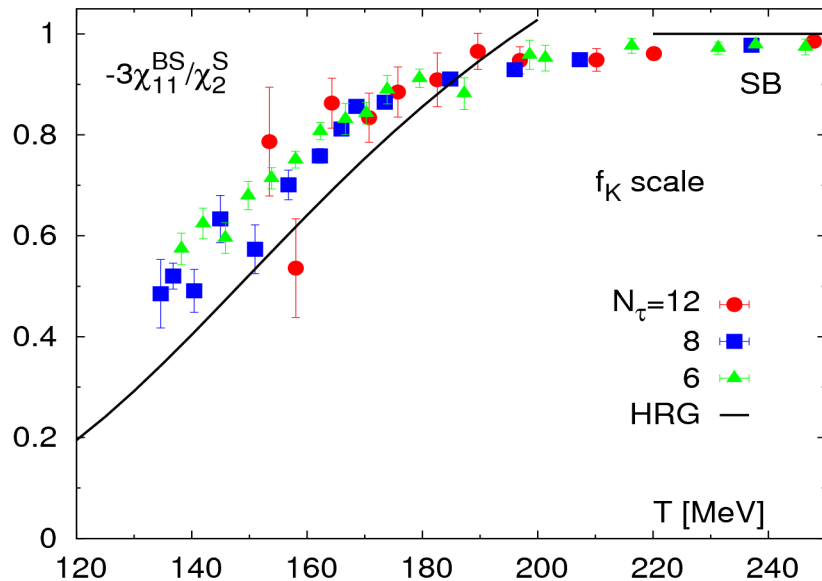- **Vanishes at high temperatures since $Q_u + Q_d + Q_s = 0$.**

# Baryon- and Charge-Strangeness
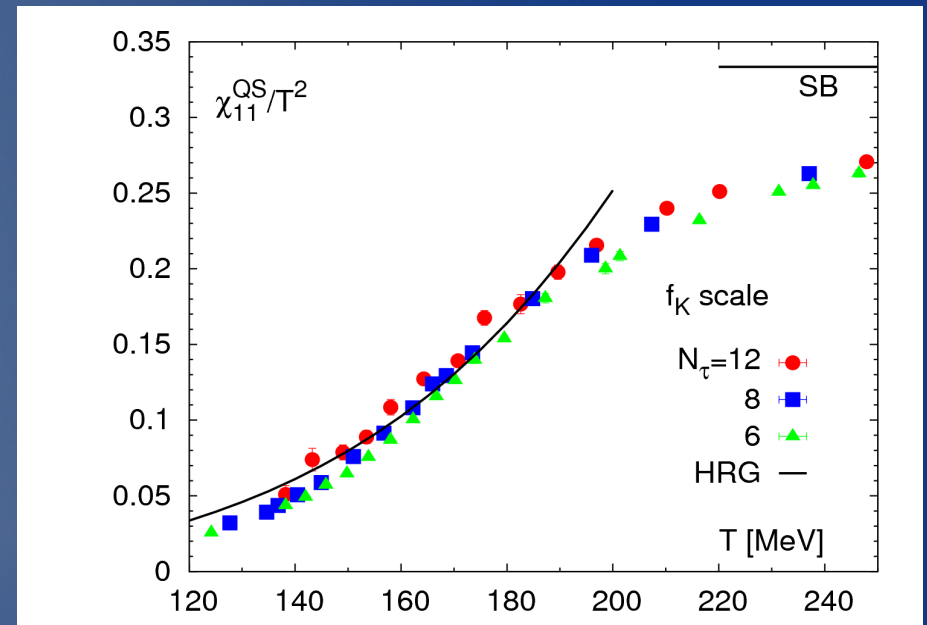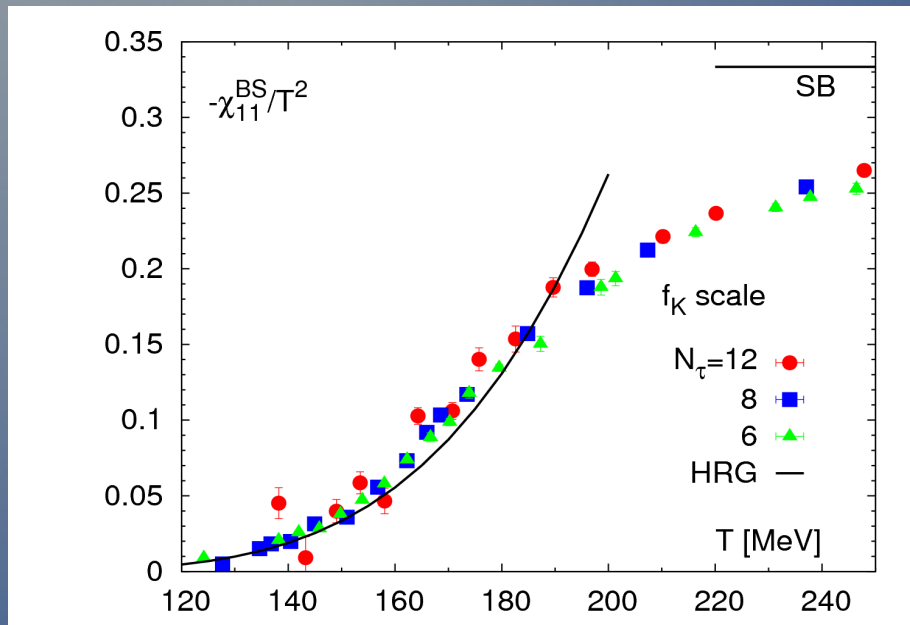


$$\langle (N_u + N_d + N_s) N_s \rangle \sim \chi_{11}^{us} + \chi_{11}^{ds} + \chi_{2}^{s}$$

- Perturbative contributions to these susceptibilities begin at $O(g^2)$ rather than $O(g^6 \ln g)$.

# Ratios: Off-diagonals to Diagonals



- These two ratios (V. Koch et al., PRL95, 182301) satisfy $-2\dfrac{\chi_{11}^{BS}}{\chi_2^S} + \dfrac{\chi_{11}^{QS}}{\chi_2^S} = 1$ at all temperatures.

- In the case of the ratios, the perturbative contributions start at $O(g^6 \ln g)$.

# Conclusions

- We have computed the lowest-order susceptibilities using a highly improved variant of the staggered action viz. the HISQ action.

- The light pion used in this study leads to a broader transition. The approach to the SB limit too is slower.

- We find good agreement with Hadron Resonance Gas models.

- Taste-breaking effects are largest in the 'Q' sector. Here continuum extrapolation is difficult and we might have to go beyond the quadratic ansatz.